# Nonuniformly Coupled Microstrip Transversal Filters for Analog Signal Processing

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*Abstract* —The Fourier transform relationship between frequency response and impedance profile for single nonuniform transmission lines [1] is used to derive the time-domain step response of single and coupled nonuniform lines. The expression for the step response of a characteristically terminated nonuniformly coupled transmission line structure is shown to correspond to the characteristic impedance profile. By using this relationship any arbitrary step response can be realized by utilizing nonuniformly coupled strip or microstrip lines for possible applications as waveform-shaping networks and chirp filters. A numerical procedure to compute the step response of the nonuniform coupled line four-port is also formulated in terms of frequency-domain parameters of an equivalent cascaded uniform coupled line model with a large number of sections. Sinusoidal and chirp responses are presented as examples that are readily implemented using coupled microstrip structures. Step response of an experimental nonuniformly coupled microstrip structure is presented to validate the theoretical results.

## I. Introduction

Nonuniform transmission lines have been used in the past to realize time-domain waveforms in various measurement systems [2]. The structures have been analyzed and designed by using traditional stepped impedance lines and cascaded section analysis based on the superposition of multiple reflections [3]. Nonuniform coupled strips and microstrip lines consisting of a large number of cascaded uniform sections have also been used as tapped delay lines, particularly at low temperatures, for broad-band superconductive analog signal processing circuits such as chirp filters [4]–[10]. For these coupled lines, step changes in coupling act as frequency selective taps with resonances corresponding to the length of the uniform coupled section, leading to a discrete time-domain response and requiring a low-pass filter to help smooth the waveform.

In this paper the relationship between step response and continuously varying characteristic impedance profile is developed for both single and coupled lines. This can be used to formulate unified design procedures for single-line wave-shaping circuits as well as the nonuniformly coupled line analog signal processing circuits. The Fourier transform relationship between taper profile and frequency response for nonuniform transmission lines [1], [11] is used together with Sharpe's equivalence principle [12] to realize a coupled line transversal filter structure suitable for waveform synthesis and signal processing. It is shown that realizability limitations are minimal, allowing for the generation of arbitrary, continuous, finite-length responses.

The applications of nonuniform coupled lines are also demonstrated by the numerical computation of the step response of representative practical chirp structures analyzed by using a CAD compatible procedure developed in the paper. In order to verify the theoretical results a set of simple experimental circuits were designed and tested. The experimental results are found to be in good agreement with theoretical predictions.

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## II. Time-Domain Analysis of Nonuniform Coupled Lines

Nonuniform transmission lines are readily analyzed in the frequency domain through Bolinder's taper profile–frequency response transform relation [1]. This analysis is extended to the time domain by identifying the elements in the transform relation that correspond to the impulse response. Upon integration a simpler step response–taper profile relation is obtained. For coupled lines, Sharpe's equivalence principle is then used to relate the step response to the even- and odd-mode impedance profile (see Fig. 1). Based on the theory of small reflections [1], [13], Bolinder's expression for the reflection coefficient in frequency domain is expressed as

$$\Gamma(z) = \frac{1}{2}\int_z^L e^{-2j\beta(u-z)}\frac{d(\ln Z_c)}{du}\,du \tag{1}$$

where $Z_c(z)$ is the normalized characteristic impedance taper profile, and $\Gamma(z)$ is the reflection coefficient along the line. Defining the reflection coefficient at $z=0$ as the transfer function $H(\omega)$, i.e., $\Gamma(z=0)\equiv H(\omega)$, leads to

$$H(\omega) = \frac{1}{2}\int_0^L e^{-2j\beta z}\frac{d(\ln Z_c)}{dz}\,dz. \tag{2}$$

For the case of constant velocity, i.e., TEM approximation, the exponential term in the integral can be expressed as

$$e^{-2j\beta z} = e^{-j\omega 2z/v}. \tag{3}$$

Letting $t = 2z/v$, then $dz = (v/2)\,dt$, $t(z=0)=0$, and

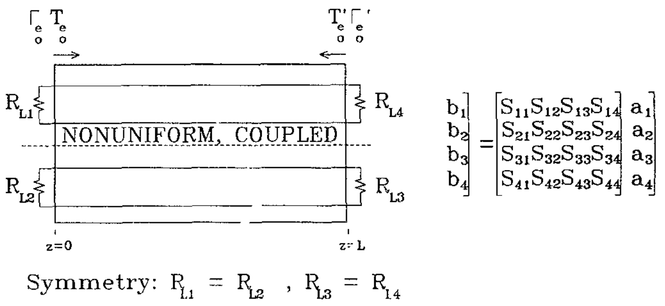


Fig. 1. Nonuniformly coupled lines as a four-port with associated $S$ parameters.

$t(z=L)=2L/v$ leads to

$$H(\omega)=\int_0^{2l/v} e^{-j\omega t}\left[\frac{1}{2}\frac{d(\ln Z_c)}{dz}\frac{dz}{dt}\right]dt. \tag{4}$$

The above integral represents the Fourier transform of the bracketed part in the integrand; i.e., the impulse response is given as

$$h(t)=\frac{1}{2}\frac{d(\ln Z_c)}{dz}\frac{dz}{dt}. \tag{5}$$

The response $g(t)$ to a unit step $u(t)$ can be evaluated by using the convolution integral. That is,

$$\begin{aligned} g(t)&=h(t)*u(t)=\int_{-\infty}^{\infty}h(\tau)u(\tau-t)\,d\tau\\ &=\int_0^t h(\tau)\,d\tau=\frac{1}{2}\int_0^t\frac{d(\ln Z_c)}{dz}\frac{dz}{d\tau}d\tau\\ &=\frac{1}{2}\int_0^{vt/2}\frac{d(\ln Z_c)}{dz}dz=\frac{1}{2}\ln Z_c\Big|_{z=0}^{z=vt/2}\end{aligned} \tag{6}$$

and finally,

$$g(t)=\begin{cases}\dfrac{1}{2}\ln Z_c\left(z=\dfrac{vt}{2}\right), & 0\leqslant t\leqslant\dfrac{2L}{v}\\ \dfrac{1}{2}\ln Z_c(z=L), & t\geqslant\dfrac{2L}{v}.\end{cases} \tag{7}$$

That is, the step response $g(t)$ or the reflection in the time domain for a step input is given by the normalized impedance profile of the taper (scaled appropriately). For the case of coupled lines, Sharpe's equivalence principle together with the expressions for the coupling coefficient of the characteristically terminated nonuniform coupled line structure can be used to express the step response of coupled lines as

$$g(t)=\begin{cases}\dfrac{1}{2}\ln Z_{oe}\left(z=\dfrac{vt}{2}\right), & 0\leqslant t\leqslant\dfrac{2L}{v}\\ \dfrac{1}{2}\ln Z_{oe}(z=L), & t\geqslant\dfrac{2L}{v}.\end{cases} \tag{8}$$

here $g(t)$ represents the even-mode reflection coefficient or the coupling coefficient $S_{21}(t)$ for the characteristically terminated coupled line four-port. This is shown in the Appendix. Note that the normalized odd-mode impedance is chosen such that the characteristic termination impedance as given by $Z_{oe}(z)Z_{oo}(z)=Z_0^2=1$.

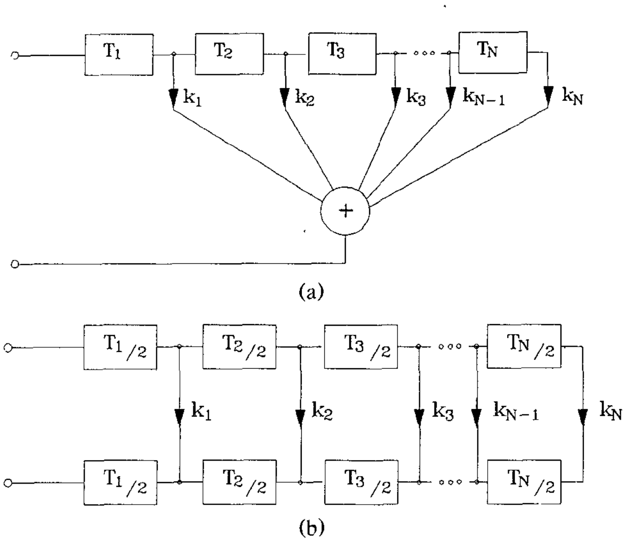


Fig. 2. (a) Transversal filter architecture. (b) An alternative transversal filter architecture.

A first-order approximation can be made for loosely coupled lines. Here, $Z_{oe}=1+\Delta Z_{oe}$ where $\Delta Z_{oe}\ll 1$. A truncated Taylor series expansion allows us to write

$$\ln Z_{oe}\cong\Delta Z_{oe} \tag{9}$$

and

$$g(t)\cong\frac{1}{2}\Delta Z_{oe}\left(z=\frac{vt}{2}\right). \tag{10}$$

That is, the variations in the even-mode impedance profile correspond directly to the time-domain step responses.

The above results are approximate and valid for loosely coupled TEM lines where (9) holds. An accurate, reliable computational procedure to evaluate the time-domain response of real nonuniformly coupled microstrip lines and striplines has also been developed, and this is used to ascertain the range of validity of the above ideal case results and to facilitate the design of the waveform-synthesizing networks and chirp filters. Here the response of general single and coupled nonuniform, dispersive transmission lines where the phase constants and impedances are dependent on the position is approximated by dividing the structure into uniform segments which are then cascaded [14]. The theory of small reflections ensures convergence to a solution as the number of uniform segments grows. For the case of coupled lines, even- and odd-mode two-port $S$ parameters can then be combined using equation (A1) (see the Appendix) to obtain the four-port $S$ parameters. The structure can in general be analyzed by multiplying the general circuit ABCD parameters of the four-port uniform sections. For symmetrical coupled lines the two procedures lead to the same results. The step response of the cascaded structure is obtained from the calculated values of the frequency-domain transfer functions.

## III. Applications and Examples

Signal processing applications for nonuniformly coupled lines range from wave shaping in digital electronics to matched filters in communications systems. Modern system requirements are continuously pushing the performance re-

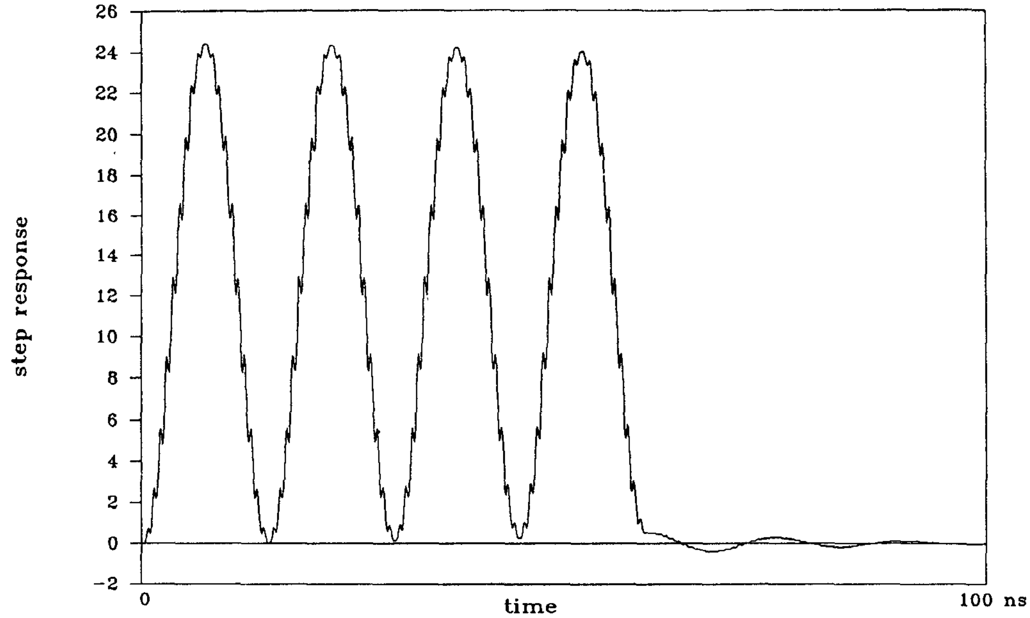



Fig. 3. Step response of a sinusoidal coupled line structure with four periods of a sinusoid; modeled with a small number of uniform segments.

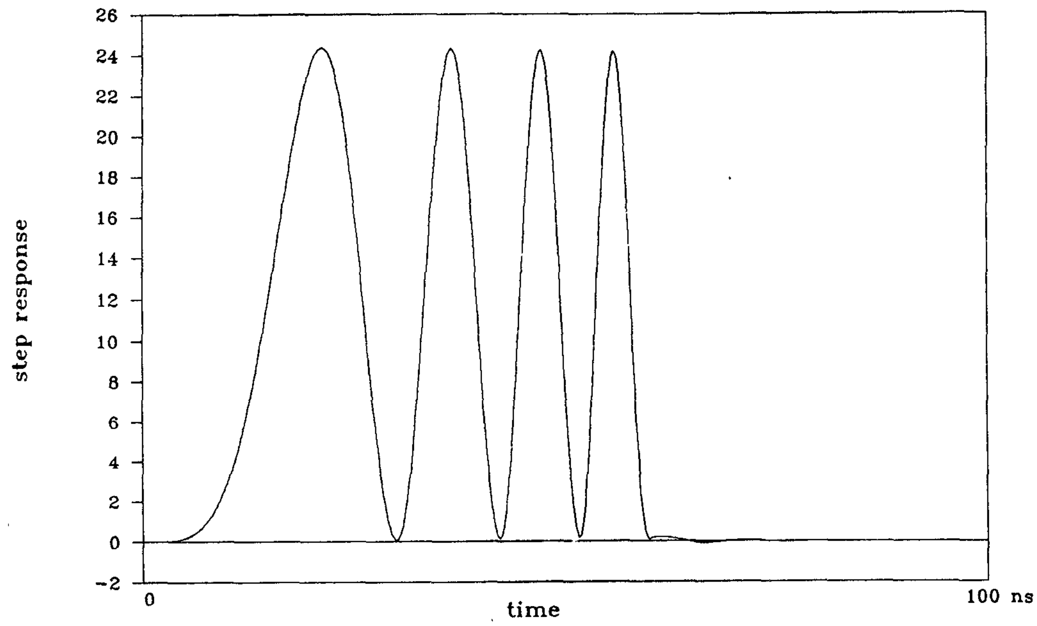



Fig. 4. Simulated response of a simple quadratic phase or 'chirp' structure.

quirements of the processing components. The emphasis in this paper is analog signal processing using transmission lines as delay elements, which have the advantage of extremely wide bandwidths. An application of interest discussed here is the linear-frequency-modulated filter. This filter is also known as a chirp filter since the step response is a linear sweep (up or down) in frequency. Such filters have been designed by using surface acoustic wave (SAW) delay lines and tapped superconducting lines. A comparison of the losses in ordinary and superconducting microstrip and SAW delay lines is given in [15] and the advantage of the superconducting line at higher frequencies is clear. The microstrip delay lines have higher loss than SAW delay lines [15], particularly at lower frequencies.

As a basis for general signal processing, the architecture for the transversal filter is shown in Fig. 2(a). The output is a weighted sum of samples taken at $T$ intervals. An alternative structure can be envisioned with delay lines in both the input and output signal paths, as shown in Fig. 2(b). A nonuniform coupled line can be thought of as a transversal filter with continuous taps. This is a limiting case, with each delay element becoming infinitesimal in length and the number of elements becoming infinite. Loose coupling is required for such structures for two reasons. Finite impulse response

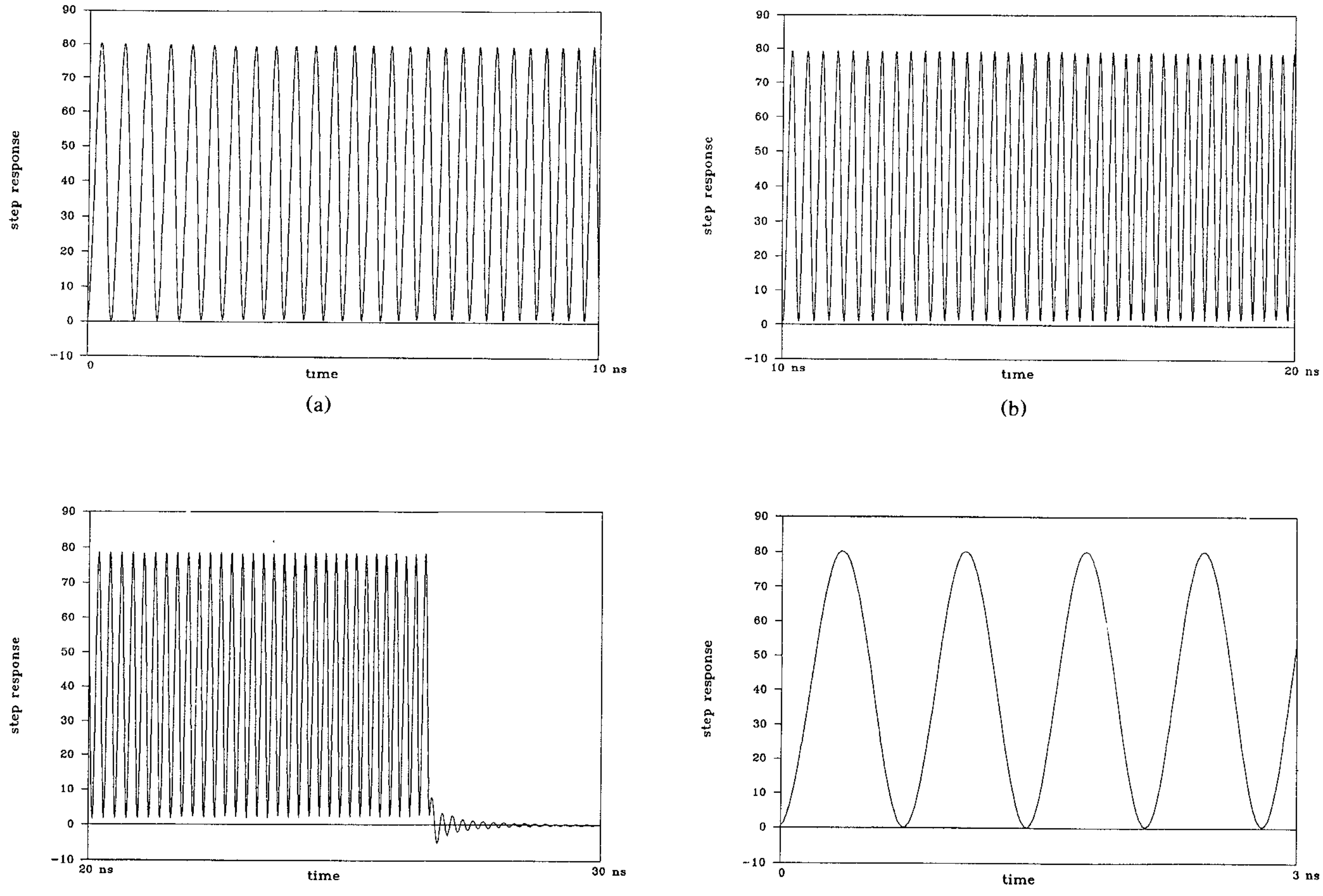



Fig. 5. Chirp of practical duration using continuous variation in coupling for 100 "periods." (a) First third of response. (b) Second third of response. (c) Last third of response. (d) Expanded view of leading edge.

behavior is desired and, hence, the secondary coupling must be kept small. Also, as an input signal travels along the line it is depleted by the fractions being coupled to the output. Loose coupling allows the approximation of constant signal along the lines.

The response of an ideal nonuniformly coupled line structure with sinusoidal taper computed by using the stepped uniform section model is shown in Fig. 3. The structure has a short sinusoidal step response and is simulated with a small number of uniform segments. The jagged regions correspond to the uniform segments. A finite number of frequency points were computed so the response is band-limited. With more frequency components the uniform segments would correspond to flat regions of the step response. As the number of segments is increased the response becomes smoother.

A simple chirp response based on the step response–impedance taper relationship as given by (10) is shown in Fig. 4. In this example the change in even-mode impedance is sinusoidal with quadratic phase as given by

$$Z_{oe} = Z_0 + \frac{(Z_{\max} - Z_0)}{2}\left(1 - \cos\left(\frac{uz^2}{v^2}\right)\right). \qquad (11)$$

The impedance variation was limited to 10% so that the approximation given by (10) is valid. This is a chirp with zero initial frequency, and the response shows an extremely broad bandwidth.

A response comparable to that demonstrated with discrete coupling in [4]–[10] for superconducting tapped delay lines is shown in Fig. 5 for continuous variation in coupling. The even-mode impedance variation is limited to 20% from the nominal 50 Ω $Z_o$. Approximately 100 "periods" of chirp over an octave bandwidth centered at 3 GHz are visible. The duration of the response is 30 ns, giving a time–bandwidth product of 60. The expanded section, Fig. 5(d) shows a very clean waveform with continuous variation in frequency and no visible distortion. A low-pass filter is not required for this case of continuous nonuniform coupling, as in the case for discrete coupling. Discrete coupling was simulated by varying the even-mode impedance in discrete steps. This results in a corresponding discrete response as seen in Fig. 6. For this case a low-pass filter is necessary to obtain the desired chirp response.

The results given above are needed as an initial design tool, but a more complete analysis including variations in even- and odd-mode impedances and phase constants with frequency as well as position is required to predict the deviation from the ideal case and help design the physical structure. Accurate closed-form equations for frequency-dependent even- and odd-mode impedances and effective dielectric constants are available for a number of propagation structures including coupled microstrips [16] and shielded strip lines. A computer program has been written which uses these equations to determine the $S$ parameters for any type of nonuniformly coupled lines, including mi-

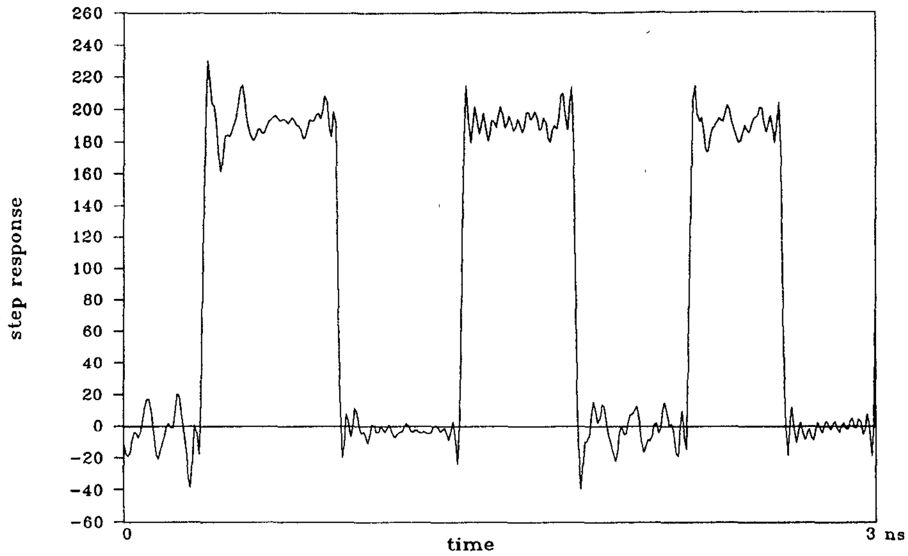



Fig. 6. Squared version of practical chirp equivalent to discrete coupling; expanded view of leading edge.

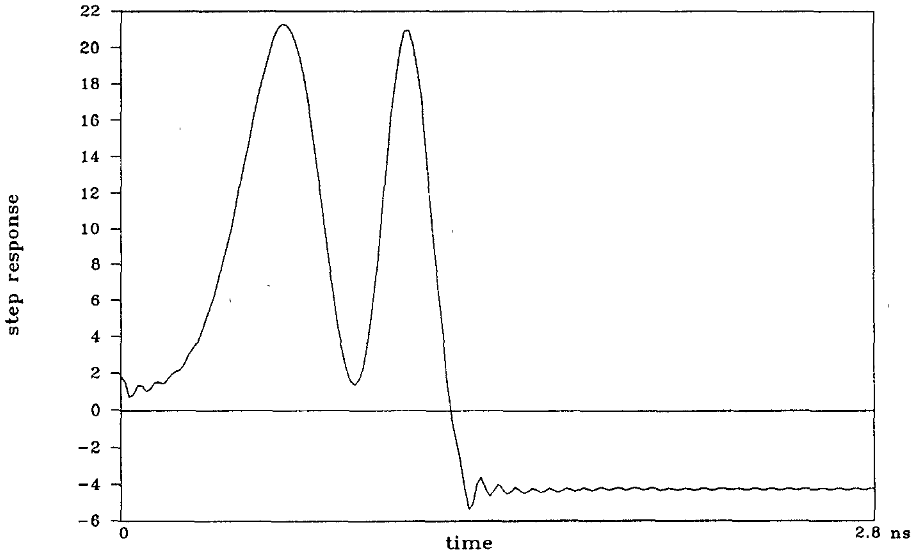



Fig. 7. Short chirp response simulated from physical dimensions of nonuniformly coupled microstrips. The simulated coupled region is 15 cm long on a 30-mil thick, $e_r = 2.2$ substrate.

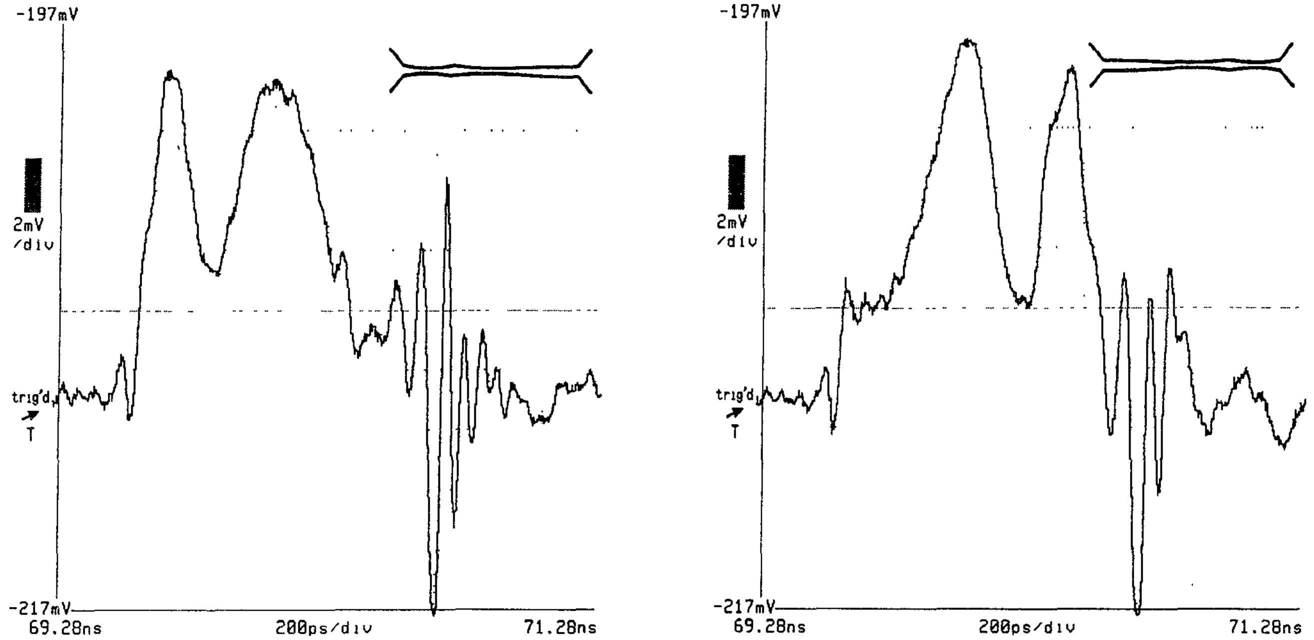
crostrips. For coupled microstrips, the width and spacing of the microstrips as a function of position are described as any number of linearly tapered sections which are specified by length, end width, and end spacing. Each linear section is divided into uniform segments for cascade analysis. Effects of velocity mismatch (between even and odd modes) and impedance variation ($Z_{oo}Z_{oe} \neq Z_0^2$) are accounted for in this analysis.

The step response for a coupled microstrip structure designed to generate a short chirp is shown in Fig. 7. For this case the deviations in the response from the design based on the ideal TEM case are minimal. Microstrip test structures were fabricated to demonstrate the validity of the step response functions presented in the earlier sections on a 30-mil-thick substrate with a relative dielectric constant of 2.20. Low-dielectric-constant material was chosen to reduce the effects of velocity mismatch. The length of the coupled line structure shown in the insets of Fig. 8(a) and (b) is 5 in.

Time-domain measurements were made using a Tektronix 11801 time domain reflectometer system. The TDR results are shown in Fig. 8. For both cases discontinuities occur at the connectors, which results in significant ringing at the tail end. For the coupled line cases the far end ports were terminated in 50 Ω. The coupled line step responses measured for up and down chirps, however, are in good agreement with the theoretical predictions.

## IV. Concluding Remarks

This approach to the design of nonuniform coupled lines presented here has been shown to be useful for synthesizing continuously varying step response functions. The initial design is based on the desired step response, which corresponds to the characteristic impedance profile for the ideal TEM case. The final design can then be based on the exact simulation of the initially designed four-port structure in-




(a) (b)

Fig. 8. TDR measurements of microstrip test structure. The coupled region is approximately 15 cm long on a 30-mil-thick, $e_r = 2.2$ substrate. (a) Down-chirp response. (b) Up-chirp response.

cluding all the dispersion effects, the losses, and the variation of the velocities and impedances with position along the structure. The usefulness and accuracy of the procedure as a design tool have been demonstrated experimentally.

The applications of structures based on the techniques presented here were limited to signal processing and waveform synthesis. Practical implementation for some signal processing applications requires low-loss transmission lines, which can only be achieved using superconductors. The design procedure for nonuniformly coupled superconducting lines is similar to the one presented here in terms of the even- and odd-mode characteristics of the superconducting structure including the kinetic inductance terms [17]. The recent advances in high-temperature superconductors suggest that more general applicability remains possible for the future. Applications under consideration using electromagnetic delay lines [4] include chirp filters used for real-time spectrum analysis and pulse compression in radar systems as well as low-phase-noise oscillators. The techniques and results presented in this paper should be useful in the analysis and design of such systems.

## Appendix
## Equivalent Nonuniform Transmission Line

Sharpe's equivalence principle [12] asserts that an equivalent pair of dual nonuniform transmission lines exists for every nonuniform directional coupler that is electrically symmetric. It is shown that for symmetric nonuniform coupled lines with constant characteristic impedance, i.e., $Z_{oe}(z)Z_{oo}(z) = Z_0^2$ constant, the even- and odd-mode characteristic lines are dual equivalent transmission lines. This enables us to describe the decoupled system in terms of the even-mode nonuniform transmission line.

Writing the defining equations for symmetrical four-port $S$ parameters (Fig. 1) in terms of the even- and odd-mode two-port $S$ parameters yields

$$[S] = \frac{1}{2}\begin{bmatrix} \Gamma_e + \Gamma_o & \Gamma_e - \Gamma_o & T'_e - T'_o & T'_e + T'_o \\ \Gamma_e - \Gamma_o & \Gamma_e + \Gamma_o & T'_e + T'_o & T'_e - T'_o \\ T_e - T_o & T_e + T_o & \Gamma'_e + \Gamma'_o & \Gamma'_e - \Gamma'_o \\ T_e + T_o & T_e - T_o & \Gamma'_e - \Gamma'_o & \Gamma'_e + \Gamma'_o \end{bmatrix} \tag{A1}$$

where the even- and odd-mode $2 \times 2$ $S$ parameters are written as

$$\begin{bmatrix} b_{1e,o} \\ b_{4e,o} \end{bmatrix} = \begin{bmatrix} \Gamma_{e,o} & T'_{e,o} \\ T_{e,o} & \Gamma'_{e,o} \end{bmatrix} \begin{bmatrix} a_{1e,o} \\ a_{4e,o} \end{bmatrix}. \tag{A2}$$

The desired structure for synthesis is that of a matched four-port, or contradirectional coupler. The condition of matching and isolation requires $s_{11} = s_{22} = s_{33} = s_{44} = 0$ and $s_{31} = s_{42} = s_{13} = s_{24} = 0$, respectively. Therefore

$$\Gamma_e = -\Gamma_o \qquad \Gamma'_e = -\Gamma'_o \tag{A3}$$

and

$$T_e = T_o \qquad T'_e = T'_o. \tag{A4}$$

For this case of matched symmetrical four-port, the $S$ matrix reduces to

$$[S] = \begin{bmatrix} 0 & \Gamma_e & 0 & T'_e \\ \Gamma_e & 0 & T'_e & 0 \\ 0 & T_e & 0 & \Gamma'_e \\ T_e & 0 & \Gamma'_e & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\Gamma_o & 0 & T'_o \\ -\Gamma_o & 0 & T'_o & 0 \\ 0 & T_o & 0 & -\Gamma'_o \\ T_o & 0 & -\Gamma'_0 & 0 \end{bmatrix}. \tag{A5}$$

The equivalent nonuniform transmission lines [12] are therefore described by the even- or odd-mode $S$ parameters.

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